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Tensor Completion in TT Format from Structured Fiber Samples

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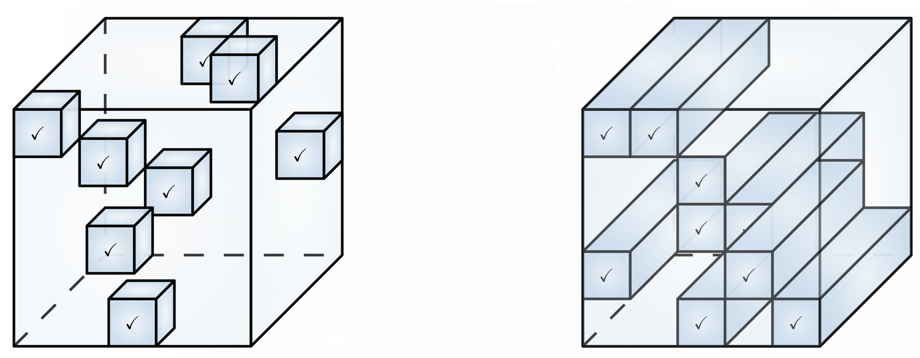
Overview: Tensor Completion

Goal: Recover a multi-way data tensor from partial observations.

Most existing methods focus on entrywise observations; here, we study fiber-structured sampling.

Entrywise random:
- Solved via (non)convex optimization.
- Probabilistic recovery conditions.
- Requires uniform sampling & incoherence.



Fiber-structured:
- Solved via standard NLA (algebraic).
- Deterministic recovery conditions.
- Fast, but slightly less accurate in noisy settings.

Tensor train (TT) decomposition

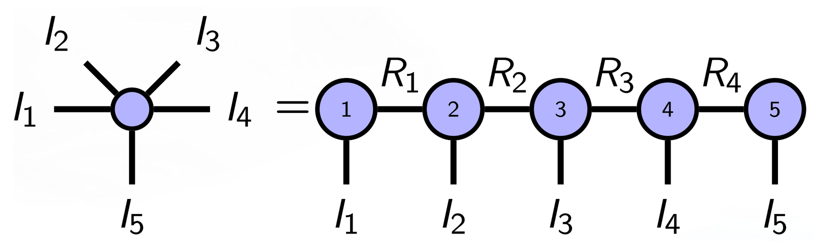
Factorizes a higher-order tensor $\mathcal{T}$ into third-order cores $\mathcal{G}^{(n)}$:

$$\mathcal{T} \approx \hat{\mathcal{T}} = \mathcal{G}^{(1)} \bullet \mathcal{G}^{(2)} \bullet \cdots \bullet \mathcal{G}^{(N)}$$

- Compression $I^N \to \mathcal{O}(2IR + (N-2)IR^2)$.

- Computable via SVD/QR when $\mathcal{T}$ is fully observed.

Example: TT decomposition for a fifth-order tensor:



Fiberwise observation in applications

1. Chemical kinetics (conc × temp × time): New conditions need new experiments; time-mode fibers are cheaper to obatin.

2. Weather time series (lat × lon × time): Full data storage is impractical; only a subset of locations observed.

Low-rank tensor completion approaches

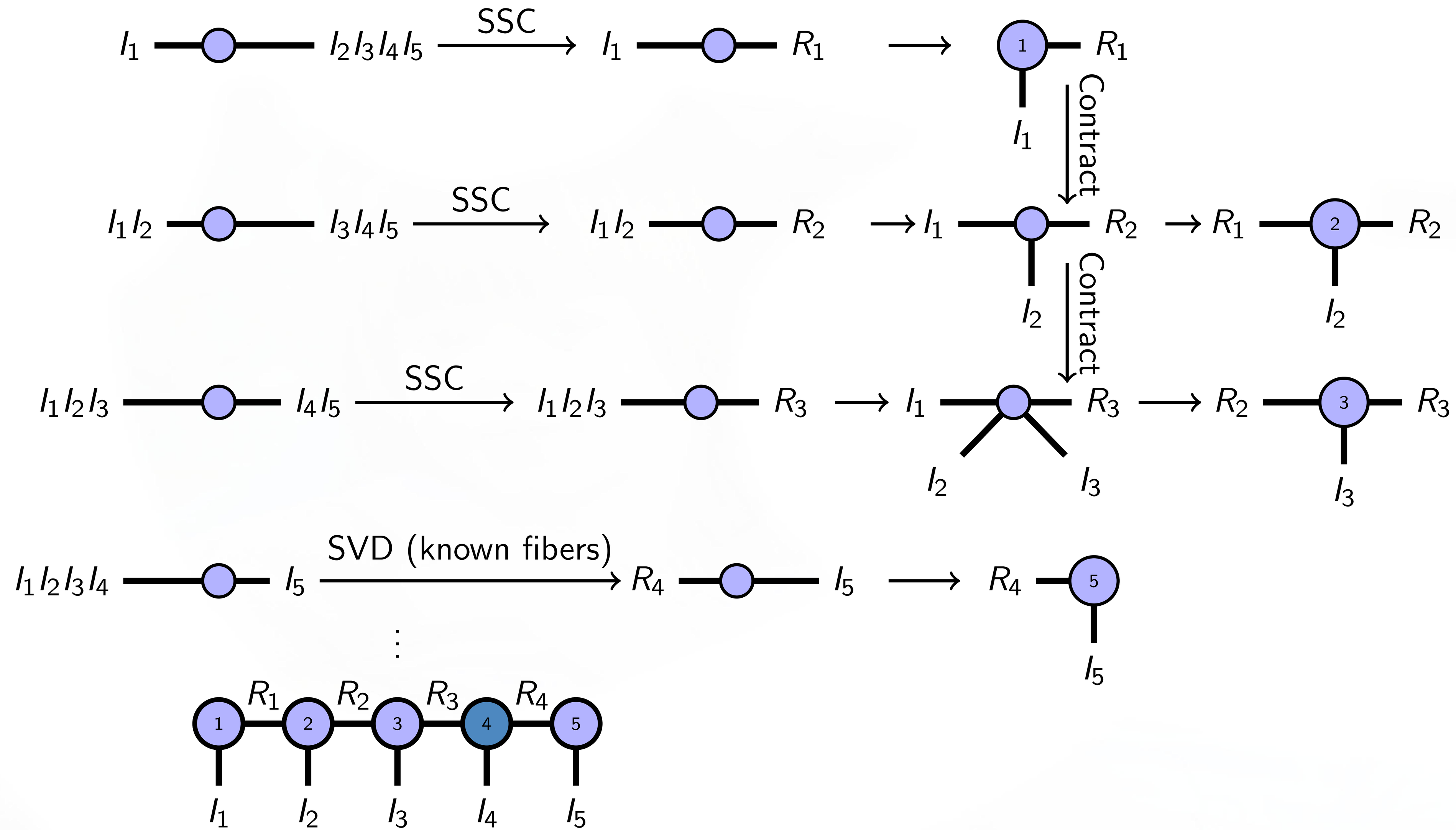
1. $\min\ \mathbf{r}_{\mathrm{TT}}(\hat{\mathcal{T}})$ s.t. $\|\mathcal{W} * (\mathcal{T} - \hat{\mathcal{T}})\|_{\mathrm{F}} \leq \delta$ (SVT, SiLRTC-TT).

2. $\min\ \|\mathcal{W} * (\mathcal{T} - \hat{\mathcal{T}})\|_{\mathrm{F}}$ s.t. $\mathbf{r}_{\mathrm{TT}}(\hat{\mathcal{T}}) = (R_0, .., R_N)$ (LS, TT-WOPT).

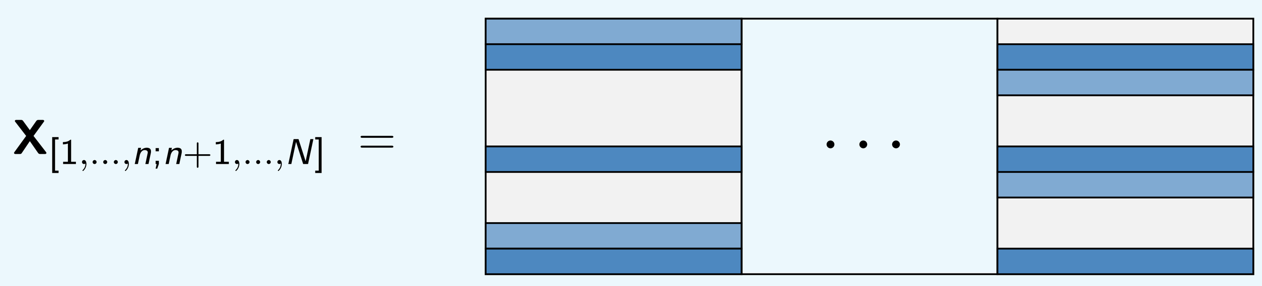
Methodology

Algorithm: TT `mode-N fibre-wise` uses SSC in place of SVD.

Consider a tensor $\mathcal{T} \in \mathbb{R}^{I_1 \times I_2 \times I_3 \times I_4 \times I_5}$ observed fiberwise along mode-5.



Subspace constraints (SSC)



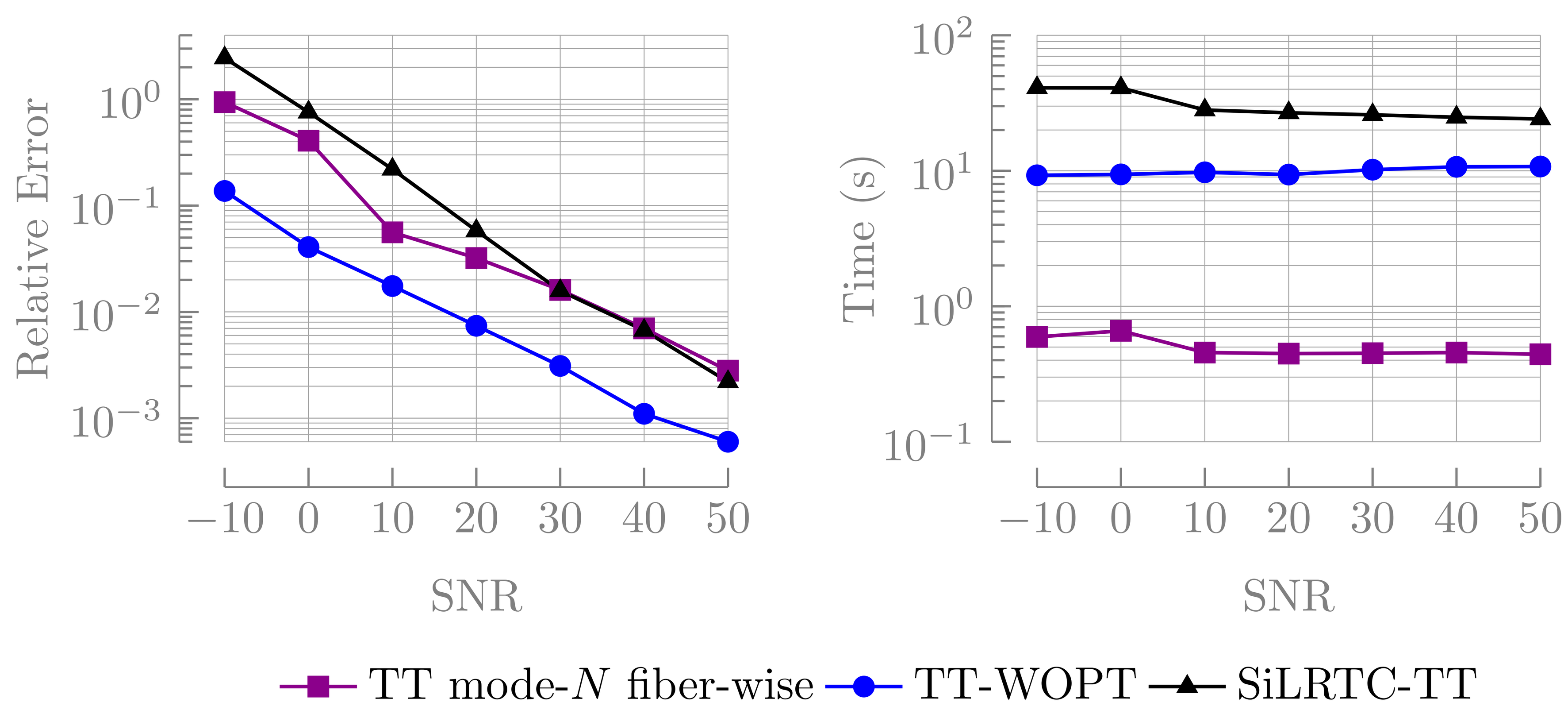
Associated subspaces: S_1 ⋯ S_K

Uniqueness condition: If $\dim\left(\bigcap_{l=1}^{L} S_l\right) = R_n$, $L \leq K$—which holds generically [1]:
- Submatrices have $\geq R_n$ observed rows.
- Each row appears in at least one submatrix.
- Some slice pairs share R_n overlapping rows.

then, $\mathrm{col}\left(\mathbf{X}_{[1,\ldots,n;n+1,\ldots,N]}\right) = \bigcap_{l=1}^{L} S_l$.

Results: completion accuracy and timing

Problem size $\mathcal{T} \in \mathbb{R}^{15\times15\times15\times15\times15}$



Conclusion

- Algebraic completion in TT format from fiber-structured samples.
- Fast, uses only standard NLA, guaranteed under reasonable deterministic conditions.
- Accurate; enables efficient optimization initialization.

References

[1]. Sofi, Shakir Showkat and De Lathauwer, Lieven (2025). *Tensor Train Completion from Fiberwise Observations Along a Single Mode.* arXiv preprint arXiv:2509.18149.