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Block Tensor Train Approach for Compressed Sensing Quantum State Tomography

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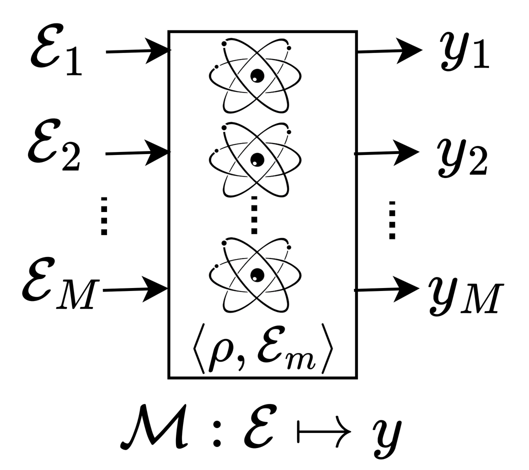
Introduction and motivation

Quantum state tomography (QST) aims to estimate the density matrix of a quantum state $\boldsymbol{\rho} \in \mathbb{C}^{D\times D}$ from measurement data $\{\mathcal{E}_m, y_m\}_{m=1}^{M}$, where $D = 2^N$ for an N-qubit system.

Key challenges:

- Exponential growth: the number of parameters scales as D^2, with $D = 2^N$.
- Physical constraints: $\boldsymbol{\rho} \in \mathcal{S} = \{\mathbf{X} \mid \mathbf{X} \succeq 0,\ \mathrm{trace}(\mathbf{X}) = 1,\ \mathbf{X} = \mathbf{X}^{\mathrm{H}}\}$.
- Scalability: standard methods are computationally and memory-intensive, becoming impractical for large systems.



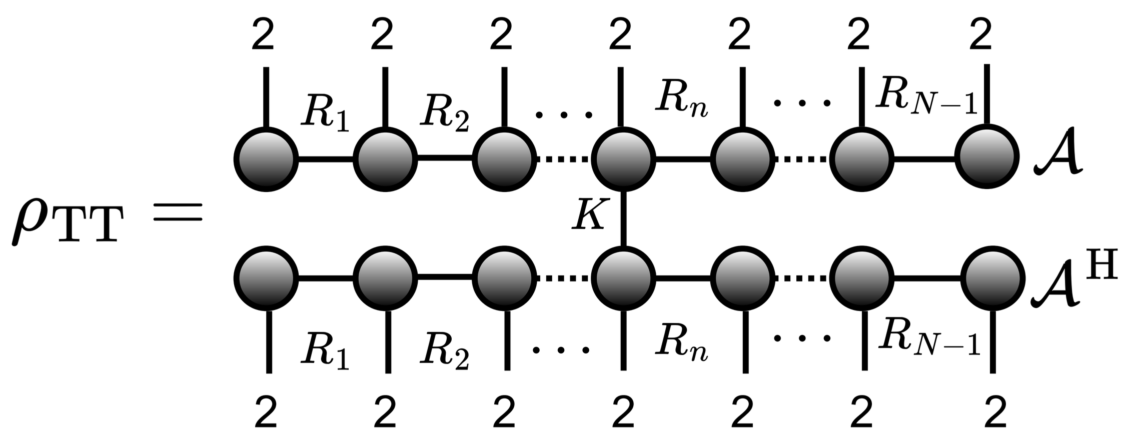
Standard compressed sensing QST

- Rank minimization: $\min_{\hat{\boldsymbol{\rho}}} \|\hat{\boldsymbol{\rho}}\|_*$ s.t. $\|\mathbf{y} - \mathcal{M}(\hat{\boldsymbol{\rho}})\|_2 \le \epsilon,$ and $\hat{\boldsymbol{\rho}} \in \mathcal{S}$ (**CVX**).
- Error minimization: $\min_{\mathbf{A}} \frac{1}{2}\|\mathbf{y} - \mathcal{M}(\hat{\boldsymbol{\rho}})\|_2^2$ with $\hat{\boldsymbol{\rho}} = \mathbf{A}\mathbf{A}^{\mathrm{H}}$, s.t. $\|\mathbf{A}\|_{\mathrm{F}}^2 \le 1$ (**LR**).

Proposed solution: Block tensor train (TT) parametrization

Mathematical model

The density operator is parameterized as a Block TT contraction: $\boldsymbol{\rho}_{\mathrm{TT}} = \mathcal{A} \bullet_n \mathcal{A}^{\mathrm{H}}$.



Key advantages:

- Physicality: Hermitian and PSD by construction.
- Memory-efficient: $D^2 \to \mathcal{O}(R^2(\log_2 D + K))$ parameters.
- Scalability: supports DMRG-style alternating optimization.
- Applicability: low-rank quantum states (e.g., pure, nearly pure, and Hamiltonian ground states).

Optimization objective

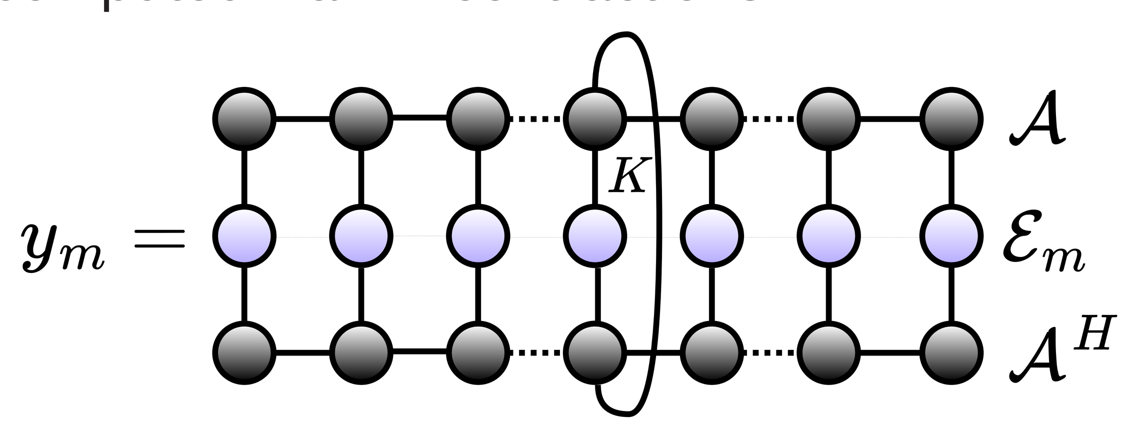
We formulate an error minimization objective using the Block TT parametrization:

$$\min_{\mathcal{A}} \tfrac{1}{2}\|\mathbf{y} - \mathcal{M}(\hat{\boldsymbol{\rho}}_{\mathrm{TT}})\|_2^2, \quad \text{with } \hat{\boldsymbol{\rho}}_{\mathrm{TT}} = \mathcal{A} \bullet_n \mathcal{A}^{\mathrm{H}}, \ \text{s.t. } \|\mathcal{A}\|_{\mathrm{F}}^2 \le 1,$$

where $\mathcal{A}$ is in Block TT format. The model parameters are optimized core-wise via (projected) gradient descent [1].

Efficient computation

Expectation values $y_m = \langle \boldsymbol{\rho}_{\mathrm{TT}}, \mathcal{E}_m \rangle$ of product-form operators $\mathcal{E}_m = \bigotimes_{j=1}^{N} \boldsymbol{\sigma}_{m_j}$, with $\boldsymbol{\sigma}_{m_j} \in \{\mathbf{I}_2, \sigma_x, \sigma_y, \sigma_z\}$ are computed via TN contractions.



- Complexity: $\mathcal{O}(D^2) \to \mathcal{O}(R^3(\log_2 D + K))$.

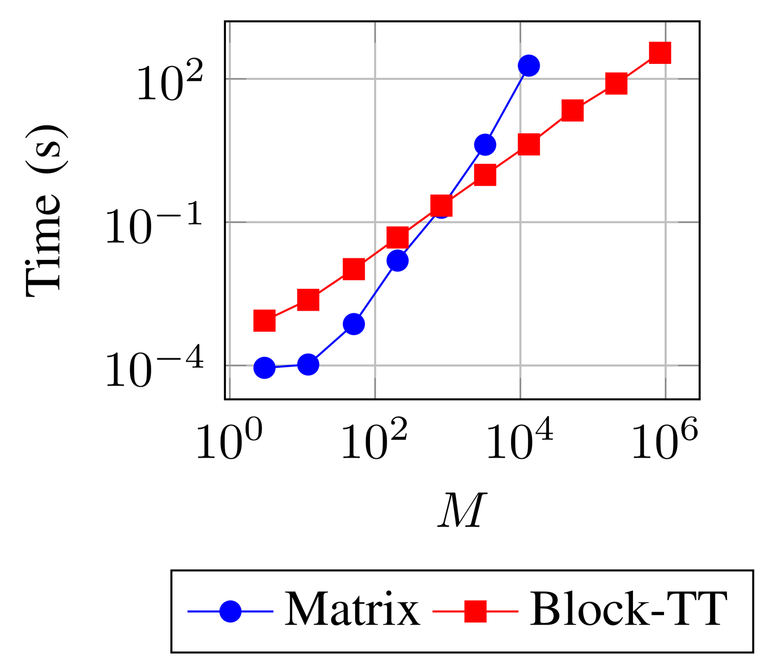


Figure 1: Measurement time for M observables scales near-linearly in the Block TT format, but exponentially in the matrix representation ($M = 0.05D^2, N = 3$ to 12).

Results

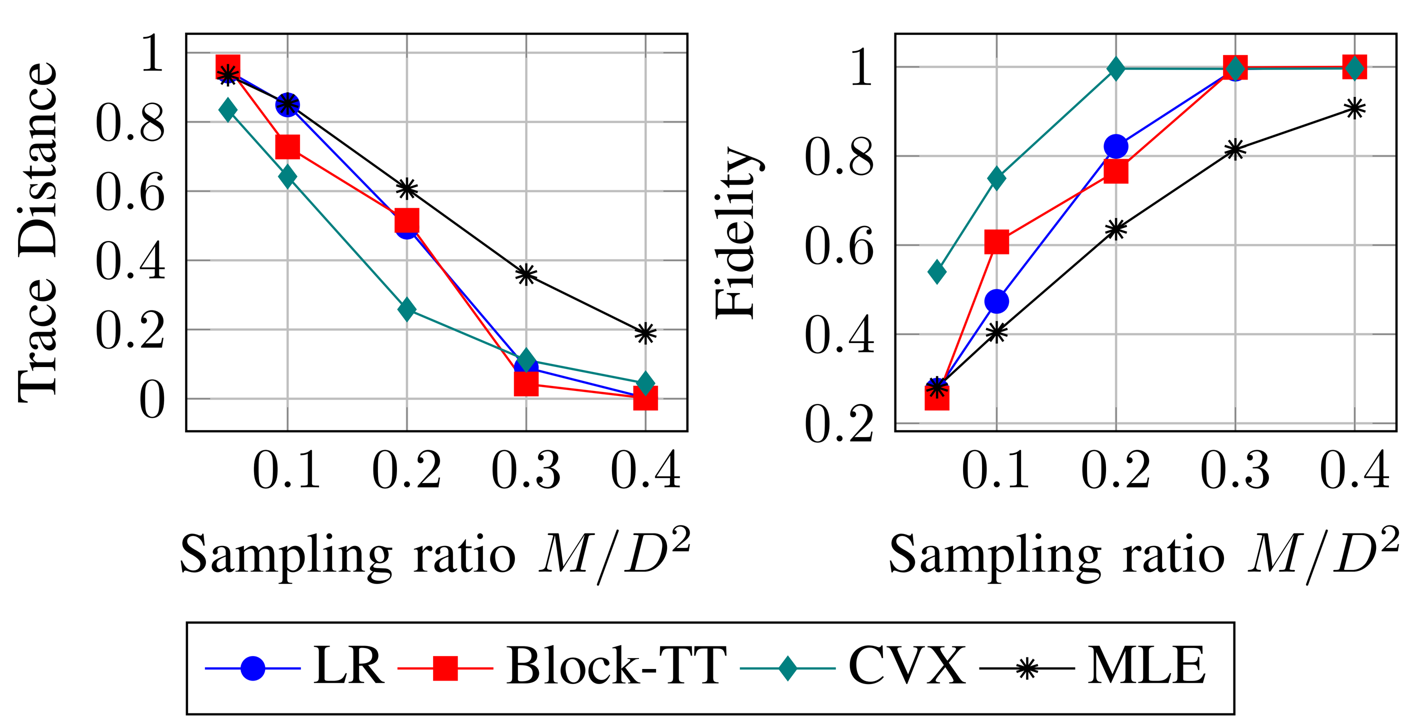


Figure 2: Our method achieves state-of-the-art accuracy in (7-qubit) mixed-state recovery with significantly improved computational efficiency (MLE denotes Maximum Likelihood Estimation; CVX and LR are noted above.).

Conclusion

- Novel QST method using Block TT parameterization of the density matrix.
- Memory- and computationally efficient.
- Accurate, physically valid results without extra constraints.

References

[1]. S. S. Sofi, C. Vermeylen and L. De Lathauwer, *Tensor Train Quantum State Tomography Using Compressed Sensing,*. 2025 33rd European Signal Processing Conference, Palermo, Italy, 2025, pp. 1332-1336.